

744

BRIEF ACCOUNT OF, and an INTRODUCTION TO,
EIGHT LECTURES,
IN THE

SCIENCE OF MUSIC,

TO BE READ AS SOON AS

Fifty Subscribers, at One Guinea each for the Course,

SHALL HAVE SENT THEIR

ADDRESS for that PURPOSE;

IN WHICH ARE PROPOSED,

TO BE DEMONSTRATED AND TO BE EXPLAINED

The Radical Sources of MELODY and HARMONY,

Deduced from the rational Principles of the Philosophy of Sounds, from Arithmetical Calculations, and from Geometrical Divisions in the Construction of Monochords, to ascertain the different Scales of the several Genera of the *Greeks* and the *Moderns*; by a clear, a concise, and an intelligible Method, different from what has been attempted before.

B. Y.

M A R M A D U K E O V E R E N D,

Organist of *Isleworth, Middlesex.*

L O N D O N.

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I N T R O D U C T I O N .

THE love of knowledge and the desire of improvement are deeply rooted in the minds of all ; yet, from situation in life, the want of the opportunities of cherishing those dispositions : or the habits acquired by adopting, unexamined, the prejudices of those we have a good opinion of : that desire and that love often lose their influence. To be enlightened, and to have the powers of the understanding extended, is not only the wish and the pursuit, but the privilege of the assiduous, both in the contemplative and in the active parts of life ; even the indolent and the ignorant have no aversion to obtain the pleasing reflections those enjoy, who have trod the mazes of science, or penetrated the recesses of nature, could they find their way through the paths that would conduct them thither, with as little trouble as taking a nap in an easy chair.

Between these two classes of the industrious and the inactive ; there are many worthy, good-intentioned, and generous, not destitute of abilities ; who, for want of a proper plan in the early part of life, or through the neglect of those who have had the care of their education, are not great proficient in the sciences : yet, impelled by an amiable disposition of mind, take a pleasure in giving encouragement to such of their contemporaries as have employed their time and attention to enlarge the bounds of knowledge, by seeking out and rewarding merit wherever they can find it. Such deserve the greatest praise ; for if the pleasure is great to conceive and to mature : to afford aid to bring forth, and to cherish, must impart to the patronizing mind, the most pleasing sensations the possessors of wealth or power can receive.

The intention of this small tract is to explain and assist the encouragers of, and the attendants on, a Course of Eight Lectures in the Science of Music, to be immediately be delivered as soon as Fifty Subscribers have sent in their names, when a room will be provided, at the West End of the Town, furnished with the proper apparatus of instruments, scales, and plans ; to which not more than One Hundred Subscribers, and none but those that are such, can be admitted, at One Guinea each for the Course.

As this is a performance on a plan never before attempted, and as, without some description more than an advertisement can impart, the design may not be understood ; an account thereof is due to the public, as well as some preliminary remarks introductory thereto.

The obscurity that prevails in the works of those that have undertaken to explain the principles of the science of music is so great, as to make many able professors sit down contented with a partial knowledge of it's radical principles ; and, indeed,

the many seeming absurdities which those writers works abound with, have discouraged the ingenious in their pursuits so much, as to make them believe that there is no possibility of assigning any reason for those principles; or that, when attained, they are not worth the pains that have been taken.

The candid in the profession that have been conversed with on this subject acknowledge the value of ascertaining such principles, but none knew the real advantages of such a research so well as the Author's late worthy friend and master Dr. *William Boyce*, whose knowledge and abilities justly entitled him to that merited situation he enjoyed many years as master of his Majesty's band of musicians; who having seen some of the calculations, was pleased not only to commend and approve of them, but gave every assistance a long-cemented friendship could hope for, to bring the same to perfection. By this means a complete theory of the sounds in music has been established, and the principles thereof fixed on a foundation so ample and extensive, as to explain and also to define most minutely all the three genera of the diatonic, the chromatic, and the enharmonic of the ancient *Greeks*, with the different species belonging to those genera; the ratios or proportions of which are determined by a series of whole numbers, unincumbered with fractions, to eighteen places in all the scales. Dr. *Boyce* has verified in his most valuable manuscript treatise of composition, purchased of his family, (wherein the niceties of modulation are discussed and explained) all the numbers to seventeen places in the diatonic intense only, extended by double flats and double sharps, with their differences. Those numbers have since been reduced to their least terms in whole numbers, and brought to fifteen places, which will give all the divisions in the present practice contained within the cycle of one octave. The process by which these were attained is part of the intended course.

There will also be given, in the course of these Lectures, an explanation of the first principles of the division of the true scale of music; its construction by major tones and limmas, or by major tones, minor tones, and major semitones; preceding which will be shewn, the difference of a major tone and a minor tone, with scales thereof, musically, geometrically, and arithmetically constructed, wherein all the lesser divisions thereof will be seen, and all these compared with a scale of commas increasing and decreasing. These will give the opportunity of shewing the uncertainties and the errors of authors since the time that *Ptolemy* wrote; many of those authors misapprehensions of the *Greeks* will be pointed out, and this theory shewn to be established on truth and reason.

The greater and lesser systems of the *Greeks* will also be explained; the *scala maxima* shewn to be rational and of use, notwithstanding it is inconcinuous. What the concinuous scale is: what was meant by the system of a fourth or tetrachord: why the *Greeks* divided their scales into tetrachords: what is to be understood by their disjunct scales: their conjunct scales: by the *synemmenons*: the remission of the acumen, and the intention of the *gravitas*: what acumen and *gravitas* mean: all this in a view not before noticed. The names of the *Greek* notes will also be explained, and found to convey different meanings than have been hitherto thought of by different commentators; some of whom have thought the term *Νητη* to mean the lowest, and others that it meant the highest, and many other new and curious observations on those names. The several characters for the notes of their scales will be also classed, methodized, and restored, freed from the errors of *Meimbomus* in his translations of the *Greek* authors, and in the diagrams of the scales of the modes, &c. found in *Alypius* and *Gaudentius*.

The errors or defects also of *Boetius*, *Glareanus*, *Galileo*, *Zarlino*, *Kircher*, *Rameau*, &c. will be shewn, and a particular notice taken of the *Tentamen novæ Theoriae Musicae*, &c. of the learned and ingenious professor *Euler*.

The ratios of *Ptolemy* and the other *Greeks* will also be considered, as restored by *Dr. Wallis*, and compared with the ratios of the above scales of whole numbers, reduced to sexagesimals to make their comparisons easy, shewing how much they differ, and where erroneous corrected.

The reason will be explained why fretted and keyed instruments disagree with the voice, or with the sounds produced by a fine performer on the violin; the reason of the temperament on the harpsichord and organ; a proposal for the best way of adjusting those instruments; hints for an improved harpsichord, to be played upon with as much ease as the present instruments with keys are, and yet to have the sharps and the flats distinct notes, and not to substitute a very discordant sound for the true one, as at present; and this in a different way from the late *Dr. Smith's* instrument, without shifting the stops.

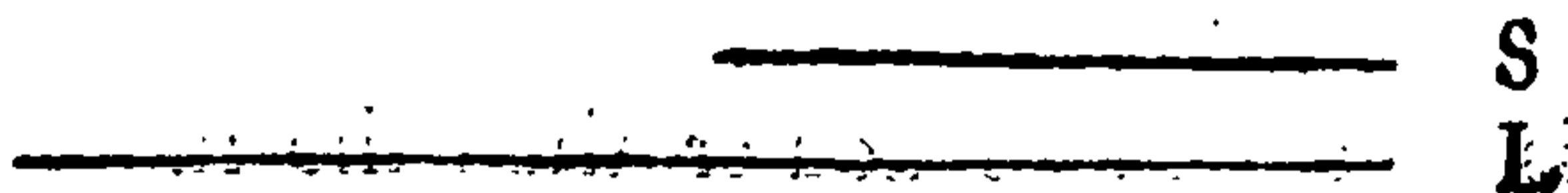
More will be added in the course; at present the following pages shew the manner of finding the ratios of the sounds by strings and by their numbers or mensurate quantity, in a way different from what these operations have been performed heretofore, the which, it is hoped, will be found an useful assistant in going through the Lectures with intelligence and satisfaction.

A Table of Intervals in Music, with their Ratios.

The radical sound or unison	1 or $\frac{1}{1}$	A major third	$\frac{5}{4}$
A major comma or simply comma	$\frac{81}{80}$	A flat fourth	$\frac{32}{25}$
The enharmonic diesis	$\frac{128}{125}$	A fourth	$\frac{4}{3}$
A semitone subminimum	$\frac{250}{243}$	A sharp fourth or tritone	$\frac{45}{32}$
A semitone minimum	$\frac{648}{625}$	A flat fifth or the semideapente	$\frac{64}{45}$
A semitone minor	$\frac{25}{24}$	A fifth	$\frac{3}{2}$
A leimma	$\frac{256}{243}$	A sharp fifth	$\frac{25}{16}$
A semitone medius	$\frac{135}{128}$	A minor sixth	$\frac{8}{5}$
A major semitone	$\frac{16}{15}$	A major sixth	$\frac{5}{3}$
An apotome	$\frac{2187}{2048}$	A flat seventh	$\frac{128}{75}$
A semitone maximum	$\frac{27}{25}$	An extreme sharp sixth	$\frac{225}{128}$
A minor tone	$\frac{10}{9}$	A lesser seventh, being the major tone inverted	$\frac{16}{9}$
A major tone	$\frac{9}{8}$	A greater seventh, being the minor tone inverted	$\frac{9}{5}$
A deficient third	$\frac{32}{27}$	A sharp seventh	$\frac{15}{8}$
A minor third	$\frac{6}{5}$	An eighth or octave	$\frac{2}{1}$

A short and easy Method of finding the Ratios of Musical Intervals by Operations, performed on Lines or Strings.

1. **I**N the following two lines, S. L. the eye instantly discovers a difference in their length.



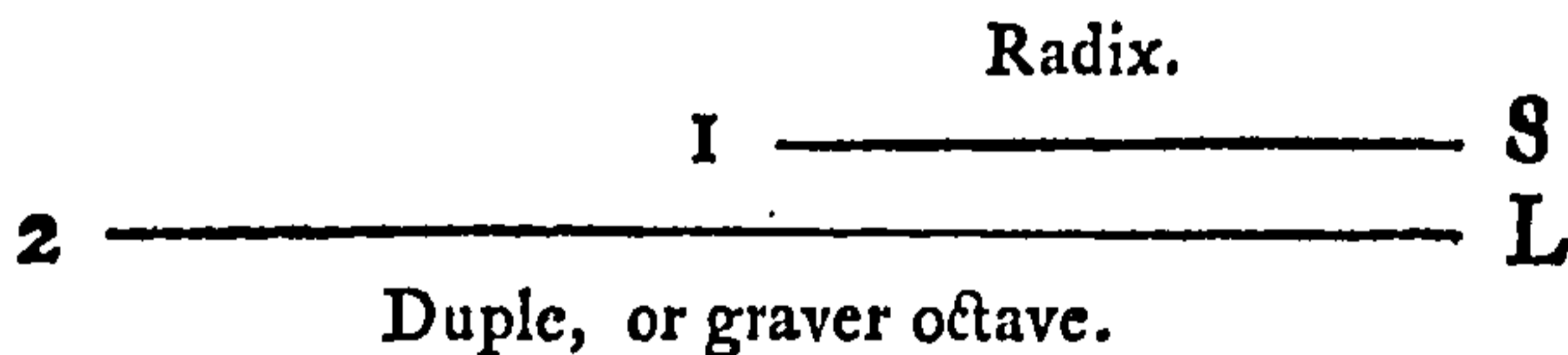
2. What that difference is may be exactly expressed by numbers, if measured with a pair of dividers; by which, if their lengths are compared, their ratios will be found to be according as the comparison is made, either in duple or in subduple ratio.

3. If S is first given to know the ratio of L to it, then S is named the radix, and L is to S in duple ratio.

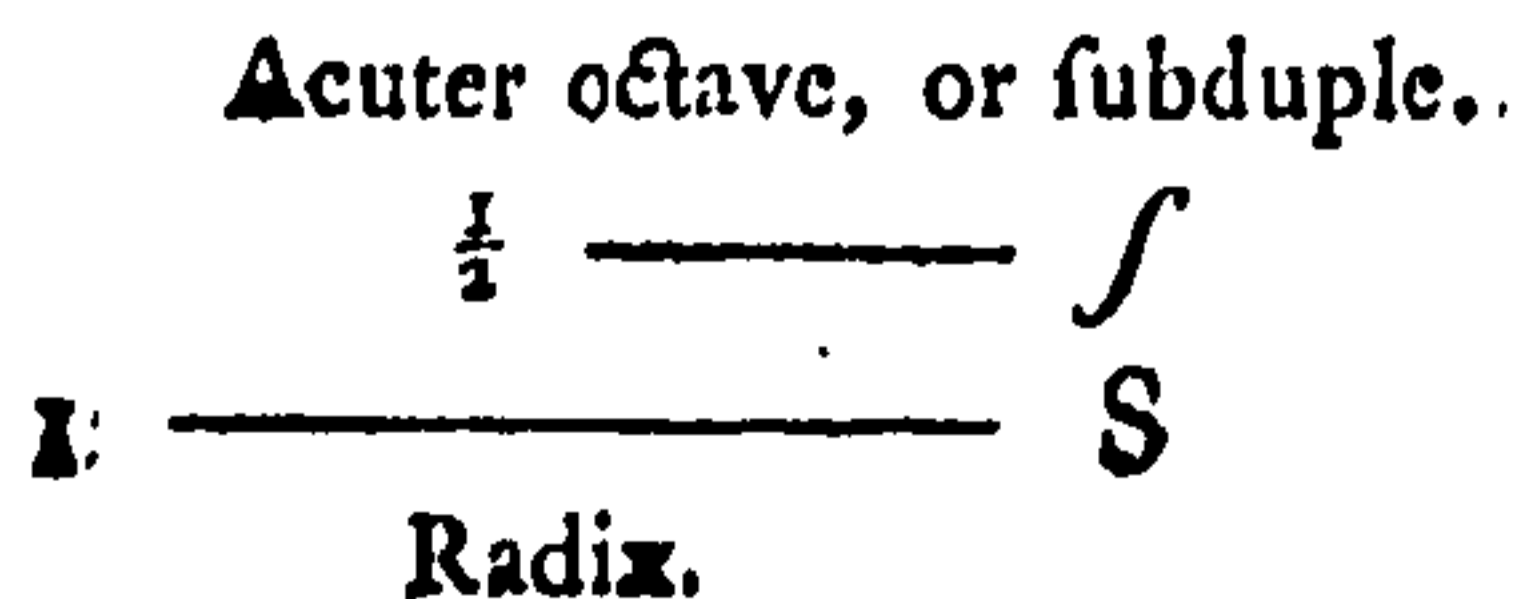
4. If L is first given to find the ratio of S to it, then L is made the radix, and S is to L in subduple ratio.

5. If these two lines were two musical strings of the same thickness or diameter, S of the length of one foot, and L of the length of two feet, and stretched by equal weights or tensions, the interval in music they would make would be, when struck and compared, the interval of an octave, estimated according to their order of sounding, 1 : 2, or 2 : 1.

6. If the sound of 1, S, is made the radix, to which the sound of 2 L is compared, that interval is the graver octave, and it's ratio $\frac{2}{1}$ is the duple, or 1 multiplied by 2.



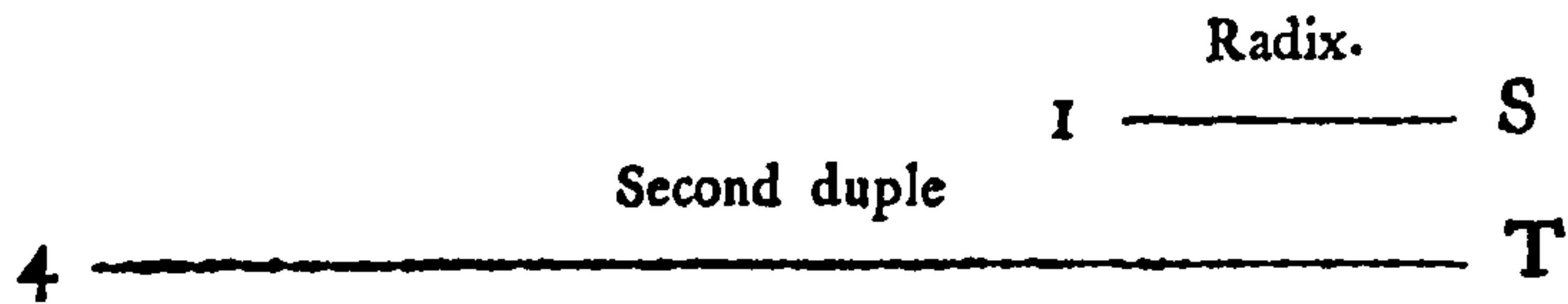
7. If 1, S, is made the radix, to which the sound of $\frac{1}{2}$ \int is compared, that interval is the acuter octave to S, and it's ratio $\frac{1}{2}$ is the subduple, or 1 divided by 2.



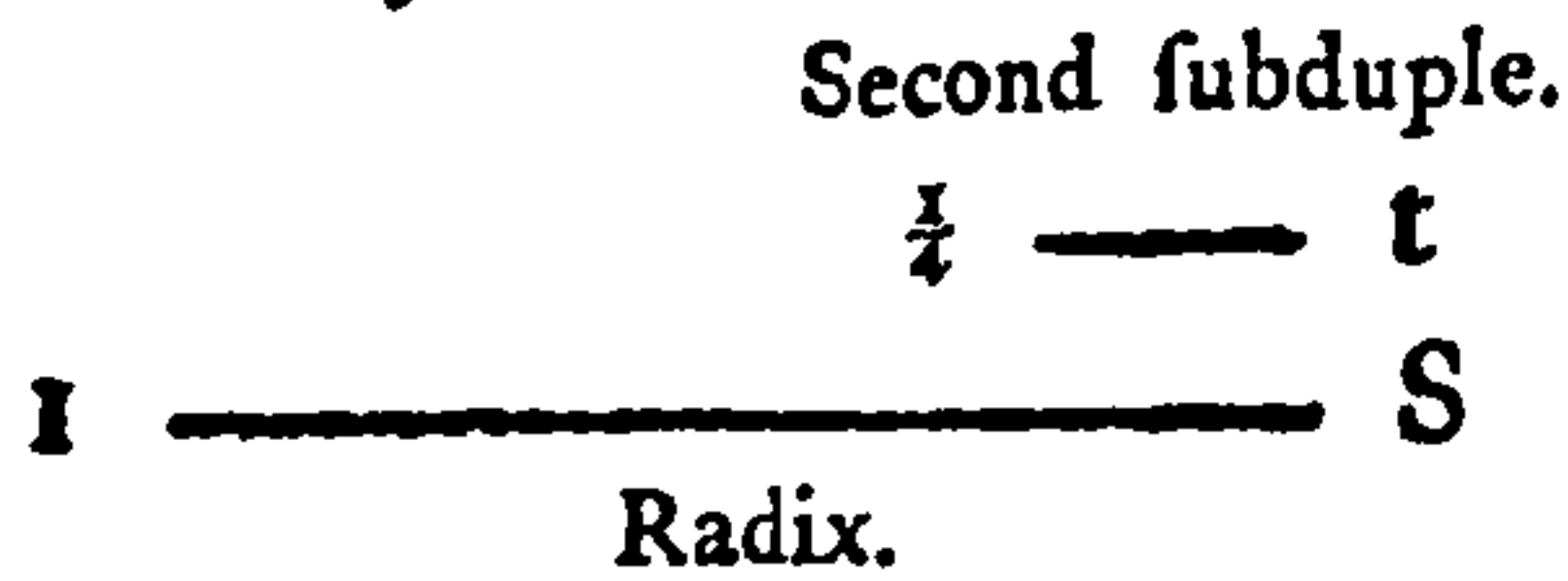
B.

8. If

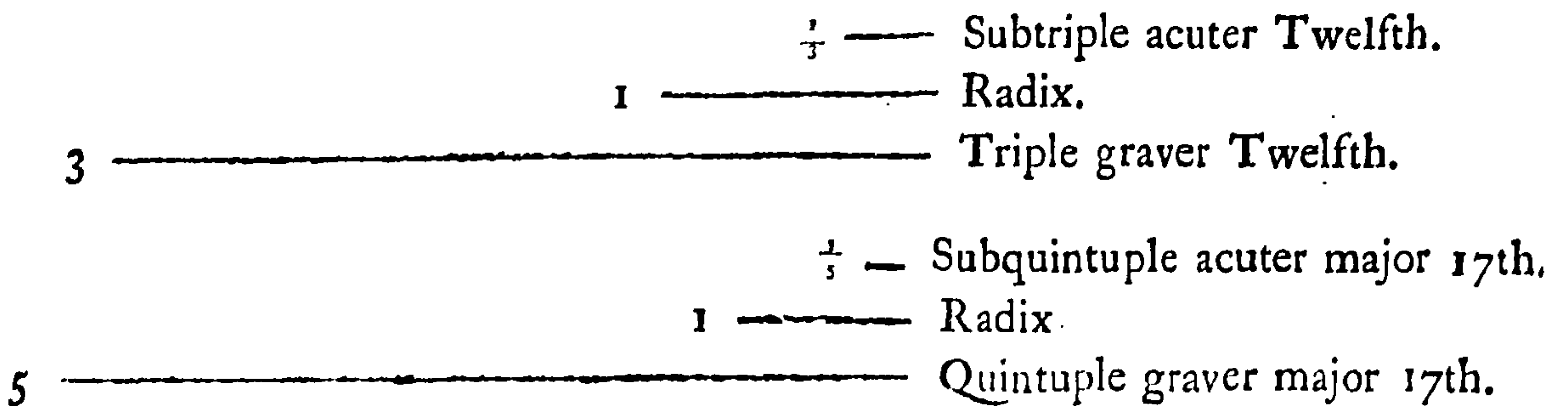
8. If two octaves, graver than the radix, are required, the first duple must be dupled again; this is named the 2d duple; it's ratio is also duple to the first duple: therefore, making 1 S again the radix, the 2d octave graver is 4 T, or $\frac{1}{4}$, being 1 multiplied by 4.



9. In like manner $\frac{1}{4}$ t is two octaves acuter than the radix 1 S, and is the 2d subduple thereof, $\frac{1}{4}$ or 1 divided by 4.

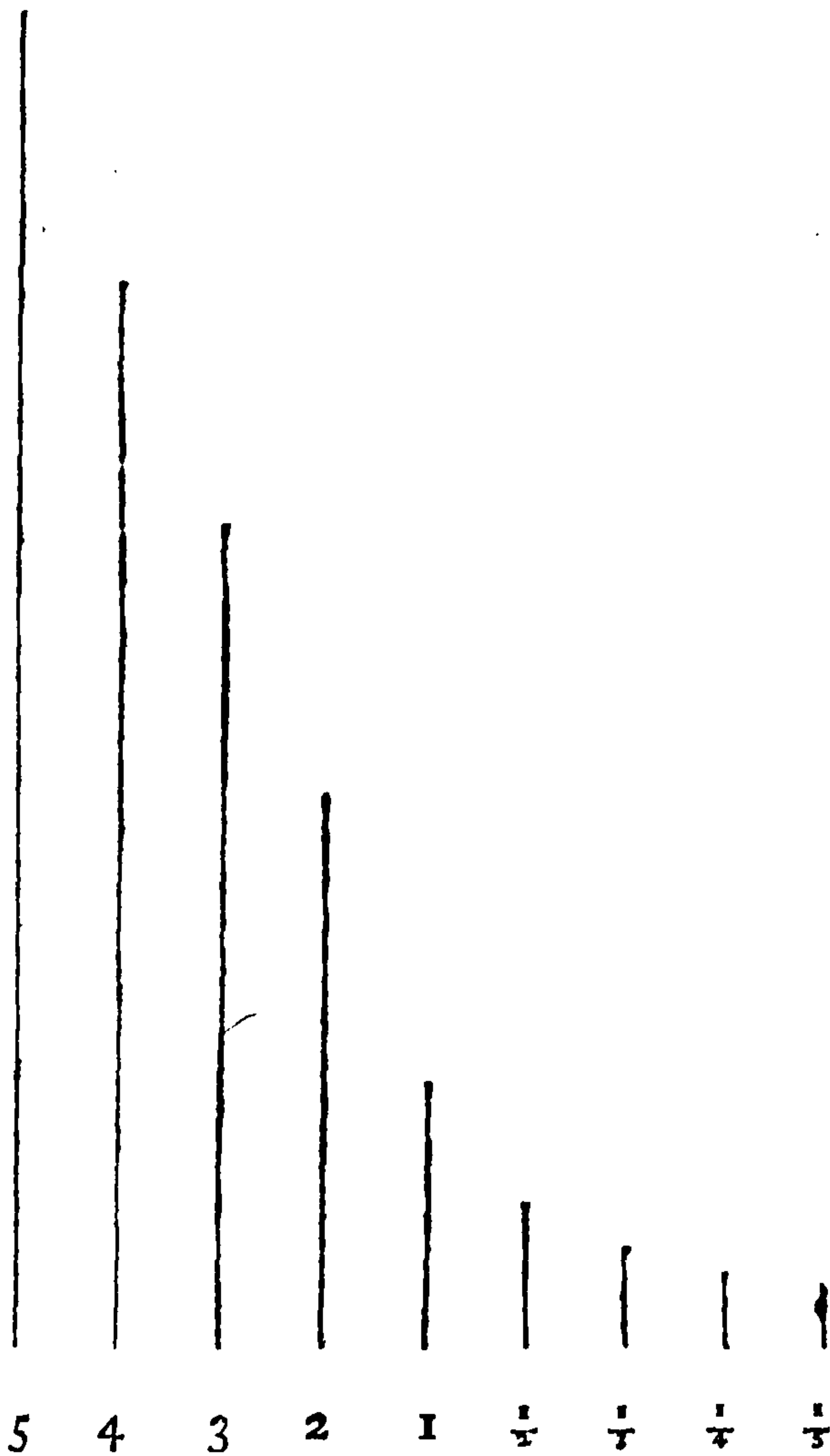


10. What has been done with the duples and subduples of 1 or unity, may be done with the triples and subtriples, the quintuples and the subquintuples thereof to infinity increasing, and to seeming infinity, though certainly finite when decreasing.



11. No other increases of the lengths of strings or decreases, but what arise from these or their various and infinite combinations, can produce sounds useful in musical composition, though many others are used in a tempered scale, and as such are borne with from imperfect instruments, although they would be condemned in a Singer or a Violinist, &c.

12. The above Lines with their Ratios collected into one View.



13. If the $\frac{1}{5}$ is made unity, the ratios below will be proportionate.

25, 20, 15, 10, 5, $\frac{5}{2}$, $\frac{5}{3}$, $\frac{5}{4}$, 1.

14. If 5 is made unity then these are like proportionals.

1, $\frac{4}{5}$, $\frac{3}{5}$, $\frac{2}{5}$, $\frac{1}{5}$, $\frac{1}{10}$, $\frac{1}{15}$, $\frac{1}{20}$, $\frac{1}{25}$.

15. If either of the above are reduced to integers in their least terms, they will be

300, 240, 180, 120, 60, 30, 20, 15, 12.

MUSICAL ARITHMETIC

16. **C**ONTAINS the definitions, the addition, the subtraction, the reduction, in order to add or to subtract; the composing, the de-composing, and the reducing to common denominators for the comparing, of the ratios by which musical intervals are expressed, whether simple, compound, or decomposed.

DEFINITIONS.

17. Simple musical intervals are of two kinds; the simple increasing intervals, and the simple decreasing intervals.

18. The simple increasing intervals in music are the graver octave 2 or $\frac{2}{1}$; the graver twelfth 3 or $\frac{3}{1}$; and the graver major seventeenth 5 or $\frac{5}{1}$, &c. These are integral ratios produced by considering unity or 1 as the radical sound, and multiplying it by 2, by 3, and by 5.

19. The simple decreasing intervals in music are the acuter octave $\frac{1}{2}$, the acuter twelfth $\frac{1}{3}$, and the acuter major seventeenth $\frac{1}{5}$, &c. These are fractional ratios found by the divisions of unity or 1, considered as the radical sound, by 2, by 3, and by 5.

20. The other musical intervals are all composed of, or de-composed by, the simple; for from the increasing or from the decreasing ratios of the simple musical intervals above defined, are all other musical intervals derived, by Addition or Subtraction, or from both.

21. Every other ratio made use of for an interval, by any author heretofore, which cannot be derived from some of the simple, or from their combinations, is not a musical number, but is an approximation, or is derived from a tempered system, and substituted for the true ratio.

22. These ratios always consist of two numbers, one of which is placed over the undermost, and named the numerator; the other number placed under the uppermost is termed the denominator.

23. The denominator shews how many parts any quantity is divided or supposed to be divided into, and the numerator how many of such divisions the ratio consists of; or the numerator may be considered as a quantity, and the denominator will express what part thereof.

24. When

24. When the greatest number is in the numerator, as $\frac{5}{4}$, it is a ratio greater than unity, formed from the simple quintuple 5, divided by 4; this gives the increased major third below the radix 1.

25. But if the lesser number of the two is in the numerator, as $\frac{3}{4}$, it is a ratio lesser than unity, found by dividing the simple triple 3 by 4, giving thereby the decreased fourth above the radix 1.

26. The same ratio $\frac{5}{4}$ may be derived from multiplying the 2d subduple $\frac{1}{4}$ by 5; and also the ratio $\frac{3}{4}$, by multiplying the same 2d subduple $\frac{1}{4}$ by 3.

27. All ratios not otherwise mentioned, are supposed to be quantities greater than unity; and when they are used in the different operations in arithmetic, are to have their largest number in their numerators.

Addition of Musical Intervals

IS performed by Multiplication of Ratios, and is the increasing of musical ratios, which may be considered as extending a musical string; to perform which this is the Rule:

28. Multiply the numerators together for a new numerator, and also multiply the denominators together for a new denominator; the first placed over the last, gives the required interval's ratio. If this new ratio is not in it's least terms, it must be reduced thereto, if any number can be found that will divide the numerator and the denominator without leaving a remainder, (such number is called a common measurer) and when so reduced the work is finished.

To add a fifth $\frac{3}{2}$ to a fourth $\frac{4}{3}$.

by	make		The increasing octave
$\frac{3}{2}$	\times	$\frac{4}{3}$	$=$
		$\frac{12}{6}$	This divide by 6)
		$\frac{12}{6}$	(make $\frac{2}{1}$
by	make		Or an octave graver

29. Or the two ratios may be first reduced, if any number can be found that will divide the numerator of the one, and the denominator of the other, without any remainder; and also if any number shall be found in the denominator of the one, and in the numerator of the other, they may be both cancelled or scratched across by a diagonal

diagonal stroke, and the remaining figures after these reductions, multiplied together, will give the required sum of the interval sought.

Thus in the same example, finding that 2 in the first will divide the numerator 4 in the second ratio twice, place that quotient 2 over the 4, and cancel the 4; and as it will divide the denominator of the first ratio once, it will be unnecessary to take notice of that 1, only cancel the 2, and the work will be thus:

$$\frac{\cancel{3}}{\cancel{2}} \times \frac{2}{\cancel{4}} = \frac{2}{3}$$

Then as in the other two places there are a 3 above, and also a 3 below, cancel them both, and the 2 above the second ratio is the required sum, under which an unite must be placed when all the figures in the denominators are cancelled. This gives the duple as before, and the whole stands thus:

$$\frac{\cancel{3}}{\cancel{2}} \times \frac{2}{\cancel{3}} = \frac{2}{1} \quad \text{The graver Octave.}$$

Another Example.

30. Add to a fourth $\frac{4}{3}$ a major third $\frac{5}{4}$ and a minor third $\frac{6}{5}$

$$\frac{4}{3} \times \frac{5}{4} \times \frac{6}{5} = \frac{2}{1}$$

Here 4 in the first and second ratios, and 5 in the second and the third ratios, cancel each the other; also 3 will divide 6 twice; put the quotient 2 over the 6, then cancel that 6 and also the 3 in the first ratio; the 2 then only being left above, shews these to make also the graver octave, as below.

$$\frac{\cancel{4}}{\cancel{3}} \times \frac{\cancel{5}}{\cancel{4}} \times \frac{2}{\cancel{6}} = \frac{2}{1}$$

More Examples of Addition.

31. Add a fourth $\frac{4}{3}$ a fourth $\frac{4}{3}$ and a major tone $\frac{9}{8}$ together.

$$\frac{4}{3} \times \frac{4}{3} \times \frac{9}{8} = \frac{2}{1} \text{ Octave.}$$

32. To add a Comma to a Comma.

$$\frac{81}{80} \times \frac{81}{80} = \frac{6561}{6400} = 2 \text{ Commas.}$$

33. To add two major tones together.

$$\frac{9}{8} \times \frac{9}{8} = \frac{81}{64} \text{ This is the superfluous third in the scala maxima.}$$

34. To add a major tone $\frac{9}{8}$ a minor tone $\frac{10}{9}$ and a major semitone $\frac{16}{15}$ together.

$$\frac{9}{8} \times \frac{10}{9} \times \frac{16}{15} = \frac{4}{3} \text{ A fourth.}$$

Here 9 in the one part cancels the 9 in the other, 8 divides 16 twice, the 2 put over the 16 and cancel both; 10 gives 2, and 15 gives 3 if divided by 5; cancel 10 and 15, then 2 multiplied by 2 produces 4 for the new numerator, and 3 in the denominator put under the 4 makes the ratio of a fourth, as above.

35. Two fourths added is

$$\frac{4}{3} \times \frac{4}{3} = \frac{16}{9} \text{ a lesser seventh.}$$

This lesser seventh $\frac{16}{9}$ added to a major tone $\frac{9}{8}$ make $\frac{2}{1}$ the octave.

Subtraction of Musical Intervals

IS performed by division of ratios, and may be considered as the diminishing of a musical string in proportion to the quantity of the proposed ratio, no alteration being made in it's tension; to perform which this is the Rule:

36. Multiply the numerator of the interval to be subtracted into the denominator of the interval from whence it is to be taken, and the product will be a new denominator; then multiply the denominator of the interval to be subtracted into the numerator of the interval from whence it is to be taken, and place the product over the first found product for a new numerator. The interval expressed by this new ratio will be the sought difference, if it is in the lowest terms, else it must be reduced by a common measurer as before.

37. Take a fourth $\frac{4}{3}$ from an octave $\frac{2}{1}$

$\frac{4}{3}$ from $\frac{2}{1}$ (leaves $\frac{6}{4}$ reduced to $\frac{3}{2}$ A fifth.

Here 4 in the first, multiplied by 1 in the second ratio, make 4 for the new denominator; and 3 in the first, multiplied by 2 in the second ratio, make 6 for the new numerator; that ratio divided in both parts by 2, gives the reduced ratio of a fifth, for the required difference.

38. The same may be reduced first, if a number can be found that will divide both the numerators without any remainder, or both the denominators without any remainder. Thus 2 will divide 4 in the first ratio, twice; place the quotient 2 over the 4, and cancel the 4; cancel also the 2 in the second ratio, as 2 is once contained in it; then, as there is no figure to multiply 3 with, carry it up to the numerator of the difference, and multiply the 2 by 1 for the denominator of that difference, and the operation will stand as below, and shew the difference to be a fifth.

$\frac{2}{3}$ from $\frac{2}{1}$ (leaves $\frac{3}{2}$ = A fifth.

39. Take

39. Take a fourth $\frac{4}{3}$ and a major third $\frac{5}{4}$ from an octave $\frac{2}{1}$

$$\frac{\cancel{4}}{3} + \frac{5}{\cancel{4}} \text{ from } \frac{2}{1} \left(\text{leave } \frac{6}{5} \text{ A minor third.} \right)$$

Here the first two ratios being to be added together before they are subtracted, are before that operation reduced by cancelling the 4 in each, then 5 is multiplied by 1 to make the denominator, and the 3 by 2 to make the numerator.

40. Take a major third $\frac{5}{4}$ and a minor third $\frac{6}{5}$ from an octave $\frac{2}{1}$

$$\frac{\cancel{5}}{\cancel{4}} + \frac{\cancel{6}}{\cancel{5}} \text{ from } \frac{2}{1} \left(\text{leave } \frac{4}{3} \text{ A fourth.} \right)$$

Here 5 being found in the numerator and the denominator of the two intervals to be added together before their sum is to be subtracted, both are cancelled; then 2 will divide the 4 and the 6 without any remainder: these being cancelled also, and their quotients multiplied cross-ways, 3 with 1, and 2 with 2, give $\frac{4}{3}$, or a fourth, for the difference sought.

41. Take a major tone $\frac{9}{8}$ from a major third $\frac{5}{4}$ and a minor third $\frac{6}{5}$

$$\frac{\cancel{9}}{\cancel{8}} \text{ from } \frac{\cancel{5}}{\cancel{4}} + \frac{\cancel{6}}{\cancel{5}} \left(\text{leaves } \frac{4}{3} \text{ A fourth.} \right)$$

Here each 5, in the ratios to be added, is first cancelled; then finding that 4 and 6 in the upper and in the lower part of those two ratios may be divided by 2, they are brought to 2 below and 3 above, and the 4 and 6 are cancelled also; then 9 in the ratio to be subtracted, and 3 in the last of the ratios from whence it is taken being both divisible by 3, put down 3 over the 9, and cancel that 9 and the 3 in the last ratio: after that 8 and 2 are divisible by 2, put down the 4 under 8, and

D

cancel

cancel that 8 and the 2. After these operations, there being nothing to multiply 3 with in the lower parts of the two last ratios, because they are all cancelled in both, place that 3 beyond all for a new denominator, and in like manner place the 4 over it for a new numerator, and that ratio is a fourth, the required difference.

The Reduction of Ratios, in order to add or to subtract, depends on this short Theorem.

42. **I**F ratios are multiplied in their numerators and in their denominators by any number, or if any number will divide their numerators and their denominators without a remainder, they will be found after either or both those operations to be in the same proportion as they were before, but in higher or lower terms.

43. Thus $\frac{1}{2}$ multiplied by 3 in both the numerator and in the denominator will make $\frac{3}{6}$, which is the same subduple in higher terms.

44. Also $\frac{24}{8}$, divided in both the numerator and in the denominator by 8, gives $\frac{3}{1}$, which is the same quantity in the least terms.

45. Further 6 : 5 : 3, multiplied by 2, make 12 : 10 : 6, equal to the above.

46. Also 16 : 18 : 20, divided by 2, give 8 : 9 : 10, in their least terms.

47. Likewise 45 : 50 : 60, divided by 5, give 9 : 10 : 12, which are proportionally the same as those quantities were before such division.

The Reduction, in order to compare two or more Ratios together, is thus performed.

48. **M**ULTIPLY the denominators of both the ratios together, if there are only two ratios : or multiply the denominators of all the ratios together, if there are more than two ratios, for a common denominator ; then multiply each of the numerators separately by all the denominators of each other ratio but it's own : these last products, placed severally over the common denominator first found, will give new ratios equal to the first given ratios, and their comparisons may thereby be clearly seen. If after these operations they can be reduced to lower terms by any number that will divide all the numerators and all the denominators, without any remainder, the result will give other ratios in the same proportion.

EXAMPLE

E X A M P L E I.

49. Reduce $\frac{1}{2}$ and $\frac{2}{3}$ to common denominators.

The denominator of the first fraction 2, multiplied by the denominator of the second fraction 3, will produce the common denominator to both, 6.

Then, the numerator of the first fraction 1, multiplied by the denominator of the second fraction 3, will produce the new numerator of the first fraction 3.

Further, the numerator of the second fraction 2, multiplied by the denominator of the first fraction 2, will produce the new numerator of the second fraction 4.

These new numerators being each wrote over the common denominator, will make these :

$$\frac{3}{6} \text{ and } \frac{4}{6} \text{ equal to } \frac{1}{2} \text{ and } \frac{2}{3}$$

E X A M P L E II.

50. Let $\frac{9}{8}$ $\frac{6}{5}$ $\frac{5}{4}$ and $\frac{4}{3}$ be compared by reduction.

First denom.	8	first num.	9	second num.	6	third num.	5	fourth num.	4
Second den.	5		5		8		8		8
	<u>40</u>		<u>45</u>		<u>48</u>		<u>40</u>		<u>32</u>
Third den.	4		4		4		5		5
	<u>160</u>		<u>180</u>		<u>192</u>		<u>200</u>		<u>160</u>
Fourth den.	3		3		3		3		4
	<u>480</u>		<u>540</u>		<u>576</u>		<u>600</u>		<u>640</u>

These placed in order over the common denominator, give

$$\frac{540}{480} \quad \frac{576}{480} \quad \frac{600}{480} \quad \frac{640}{480}$$

Equal to $\frac{9}{8}$ $\frac{6}{5}$ $\frac{5}{4}$ $\frac{4}{3}$

51. These may be all reduced into their least terms by dividing each member of the whole by 4, which, when done and compared with unity or $\frac{1}{4}$, the radix, (by multiplying it in the numerator and in the denominator by the reduced common denominator) give the under-placed ratios with their intervals or differences between each other.

Radix	$\frac{120}{120}$	tone major	$\frac{135}{120}$	femitone major	$\frac{144}{120}$	femitone minor	$\frac{150}{120}$	femitone major	$\frac{160}{120}$
Sounds	G		F		E		E flat		D

52. By similar operations are ratios also reduced to whole numbers, for if the numerators of the last reduced ratios are taken without their common denominator, they will give a series of whole numbers in the same proportion as the ratios at first were; the first term of which series will be that common denominator for the radix.

$$\text{Radix } 120 : 135 : 144 : 150 : 160$$

53. These may also be easily converted into numbers with decimal parts, or decimals of 1, 2, 3, &c. places, thus:

12 . 0	:	13 . 5	:	14 . 4	:	15 . 0	:	16 . 0	mixed
1 . 20	:	1 . 35	:	1 . 44	:	1 . 50	:	1 . 60	mixed
. 120	:	. 135	:	. 144	:	. 150	:	. 160	decimals
. 0120	:	. 0135	:	. 0144	:	. 0150	:	. 0160	decimals

Each of these are a series alike proportional, being made severally a tenth, an hundredth, a thousandth, and a ten thousandth less than the first series of whole numbers.

54. If either unity or a whole number is made use of in any of the above operations, it must be expressed as a ratio by putting 1 under it thus, as $\frac{1}{4}$ or $\frac{4}{1}$, for 1 and 4, &c.

COROLLARIES.

C O R O L L A R I E S.

55. **E**VERY ratio having 1 or unity for it's denominator is a whole number, as $\frac{1}{1}$ is 1, $\frac{2}{1}$ is 2, $\frac{3}{1}$ is 3, &c.

56. Every whole number is always supposed to have 1 or unity under it for it's denominator, to assist the understanding in having a clear idea thereof, thus, 6 is $\frac{6}{1}$, or six quantities, one of which such quantities is conceived and determined in the mind, as six shillings, six yards, or six days, &c.

57. Every ratio whose numerator can be exactly divided without leaving any remainder by its denominator, and that being more than 1 or unity, is also a whole number wrote as a ratio, but not in it's lowest terms. *Such are improperly named by arithmeticians, improper fractions, for the term improper must convey to the mind a false or confused idea.*

58. All other ratios having their numerators greater than their denominators, being not exactly divisable thereby, and their denominators greater than 1 or unity, are mixed numbers or quantities greater than a whole number, being an unit or a whole number and a fraction, as $\frac{6}{5}$ gives $1 \frac{1}{5}$; $\frac{9}{2}$ is $4 \frac{1}{2}$; and $\frac{9}{4}$ is $2 \frac{1}{4}$. *These are also improperly named improper fractions.*

59. Any ratio having in it's numerator 1 or unity, and in it's denominator some number greater than unity, but which cannot be divided by any other number than unity, is a simple fraction, as $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, &c.

60. If the denominator of any fractional ratio, having unity for it's numerator, can be divided by any other number besides unity, it is a decompound fraction, $\frac{1}{4}$, $\frac{1}{6}$, $\frac{1}{9}$, &c. The first may be divided by 2, the second by 2 and by 3, and the third by 3.

61. Every ratio in it's least terms having a number greater than unity in it's numerator, and it's denominator greater than the numerator, is either a compound fraction, or else a decompound fraction, according to the manner of it's production; it may also be a compound and a decompound at the same time.

62. The compound fraction $\frac{3}{5}$ is made up of three, fifth parts of an unit, if $\frac{1}{5}$ is given to multiply it by 3.

63. The same $\frac{3}{5}$ may be termed a decompound fraction, if 3 or $\frac{3}{1}$ is given to be divided by 5; it is then in that case decompounded, or understood as the fifth part of the quantity 3.

64. Further, the same $\frac{1}{5}$ may be both a compound and a decomposed fraction, if found by having $\frac{1}{7}$ given to multiply it by 3, and afterwards to divide that product by 5, this is an unit first increased three times and afterwards decreased a fifth part.

65. The idea of compound, is an increase of quantity.

66. The idea of decomposed is a decrease of quantity.

67. Hence the fraction of a fraction, as the $\frac{1}{2}$ of $\frac{1}{4}$ of $\frac{1}{5}$ is a decomposed fraction, and not a compound fraction, as has always been heretofore taught; for the $\frac{1}{5}$ of an unit is thereby made the $\frac{1}{100}$ part thereof.

68. If a ratio or a fraction is to be multiplied by a number, and that number will evenly divide the denominator of such ratio, the quotient of such division placed under the numerator will give a ratio equal to the first ratio, if it's numerator was multiplied by such number, and afterwards reduced to its least terms, which is the manner of doing it, if the number will not evenly divide the denominator. Thus, $\frac{2}{9}$ multiplied by 3 makes $\frac{6}{9}$, whether the 2 is multiplied by 3, or the 9 divided by 3; in the first case, there will be found $\frac{6}{9}$, which reduced is $\frac{2}{3}$, as before.

69. If any fraction or any ratio is to be divided by a number, and that number will evenly divide the numerator of such ratio or fraction, the quotient of such division placed over the denominator will give a ratio equal to the first ratio, if it's denominator was multiplied by such number and afterwards reduced to it's least terms, which must be done if that number will not evenly divide the numerator. Thus, $\frac{8}{9}$ divided by 4 gives $\frac{2}{9}$; whether 8 is divided by 4 first, or whether 9 is multiplied by 4, making $\frac{36}{9}$, and afterwards reduced to it's least terms, it gives $\frac{2}{9}$, as above.

70. All numerators are real multipliers or increasers, as $\frac{1}{2}$ or $\frac{3}{4}$, &c.

71. All denominators are real divisors or decreasers, as $\frac{1}{2}$ or $\frac{1}{3}$, &c.

What is here delivered has produced easy solutions to a vast number of abstruse and controverted questions on the divisions of the scales of music; and several problems have and may again be wrought in a few minutes, by the assistance of some harmonical tables constructed thereby, as would in the common way of calculation require several years for their performance, even if no error should occur in the operation that way, which it is hardly possible to expect, for frequently the numbers are 60, 100, or more places, before their reduction to their least terms.

As an instance of what is here advanced, and to shew the brevity of such method, are given the following ratios, which are the quantities that are contained in the compass of one octave of the scale of sounds belonging to the species of the diatonic intense, in both the acumen and in the gravitas, with the reduced extensions to double flats and double sharps; all which the late Dr. *Boyce* verified from four radical numbers first ascertained and imparted to him for that purpose.

Within the extremes of such an octave are contained all these differences between it's several component parts.

The number of the ratios, and the nominal quantities of each.

Four of $\frac{10737418240}{10460353203}$ An enharmonic diesis and a minor residual.

Two of $\frac{70368744177664}{68630377364883}$ Two minor commas and a minor residual.

Six of $\frac{858993459200}{847288609443}$ A minor comma and a minor residual.

Sixteen of $\frac{81}{80}$ A major comma.

Nineteen of $\frac{2048}{2025}$ a minor comma. And

Thirty-eight of $\frac{32805}{32768}$ A schisma.

The above make 85 differences between 86 sounds, which are the number of subdivisions in that octave. When they are all summed up and reduced, they make $\frac{2}{7}$.

They have been, and at any time may again be added, by any person only acquainted with the first two rules of arithmetic, in less than ten minutes.

A minor residual is $\frac{10485760000}{10460353203}$

The pleasure of having made this discovery is a sufficient reward for the application and labour bestowed on the research, even if no other advantage should be derived from the same.

CONCLUSION.

C O N C L U S I O N.

EVERY thinking enquirer after improvement, to whose hands this may happen to make it's way, must perceive the great advantages to be derived from an investigation of these truths. For, who so shall be able to class and arrange methodically and clearly; to mark down and to point out, the divisions and the distinctions of all the sounds in melody and in harmony, belonging to the scales of the different species of all the genera, in music, accurately and also evidently unto the senses and unto the reason; to ascertain the component parts thereof, their ratios and their connections with their differences exactly and minutely; and by these means to fix in the mind indisputably and indubitably, that beautiful order in which the principles of this DIVINE SCIENCE have always existed, still remain unchanged, and eternally shall endure — Must have as much advantage above another not so informed, as one voyager with an exact chart and compass must have over another assisted by repeated trials and experience only, notwithstanding the knowledge of the latter may be thought and likewise be found to be sufficient.

T H E E N D.

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